

Steady State and Response Characteristics of a Simple Model of the Carbon Cycle

BERT BOLIN

ABSTRACT

A model of the carbon cycle with seven reservoirs (atmosphere, mixed layer of the sea, deep sea, short-lived terrestrial biota, wood, humus and soil) is developed and its steady state and response characteristics are analyzed. A slow exchange between the two ocean reservoirs and turn-over time for the deep sea of about 2000 years, yields a ^{14}C distribution between the atmosphere and the sea that is consistent with the observed slow rate of sedimentation in the deep sea 5000–10 000 years ago as compared with the present rate and also explains the discrepancy between radiocarbon age and true age about 5000 years ago. The response characteristics of the model are deduced for both exponential and periodic external perturbations. The oscillations of the carbon cycle due to periodic variations of the sea surface temperature are analyzed.

1. SCOPE OF THE PROBLEM

A number of models of the carbon cycle have been developed in recent years with the aim to understand the mechanisms that determine its main characteristics and to assess the way man is modifying it, by fossil fuel combustion and exploitation of the terrestrial biosphere. It is obviously important to validate any such model with the aid of real data and particularly to try to determine what we know with some certainty about the cycle and what we do not because of too simplified models or inadequate data for validation. Not until this is attempted systematically, and some features of the carbon model have been firmly established in this way, will we be able to proceed further with more advanced models. Such work is also important for determining what kind of data are most relevant for the validation of more advanced models and what degree of accuracy is required.

The present article will not deal with this problem as a whole, but rather we shall consider some special aspects of it, which will show that uncertainties are still considerable and that data are much needed for improving the present situation. Obviously more resolution in models is highly desirable, since it otherwise is difficult to identify model variables with measured variables; this makes a validation less reliable.

More detailed models will require the identification and analysis of subsystems

and their characteristic dynamics, which also require proper data. Presently available simple models of the carbon cycle should be used particularly for identification of those parts of the carbon cycle which warrant more detailed treatment to resolve present uncertainties. It is then equally important to determine what expansion of the data base is necessary to permit the proper inclusion of this more detailed treatment of crucial parts of the carbon cycle.

2. A SIMPLE MODEL FOR THE CARBON CYCLE

For the following analysis we shall adopt a simple 7-reservoir model for the carbon cycle as depicted in Figure 1. The oceans are described by merely two reservoirs: one for the surface layers and one for the remainder of the sea. It might be desirable to consider more detailed formulations as advanced by Björkström (1979) and particularly to distinguish between warm and cold surface water and to account for the role of the thermocline region or intermediate water. For the particular aspects of the problem which we wish to consider here, this simple model should be adequate. This assertion, however, needs validation by the use of more complex models. It should particularly be noted that the biological processes in the sea have not been described explicitly, but as Keeling (1973) has shown they can implicitly be taken into account in an approximate manner in the determination of the exchange rates between the two ocean reservoirs being considered (see further below).

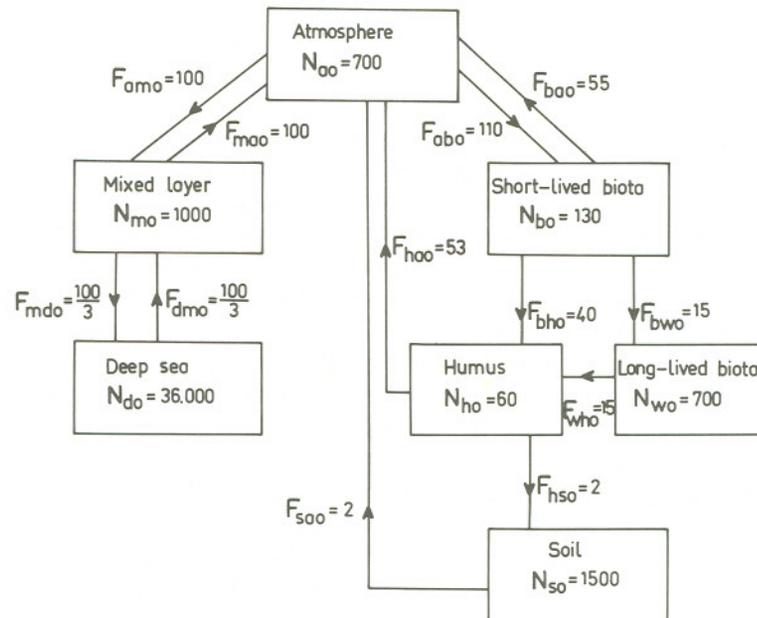


Figure 1. Seven-box model of the carbon cycle. Reservoir sizes are given in 10^9 ton and the transfers between them in 10^9 ton yr^{-1} .

The terrestrial biota are described in terms of merely one ecosystem with four major components: (b) shortlived biota active in assimilation and growth (leaves, needles, fine roots etc); (w) long-lived biota which form the living structures (wood, bark, branches, etc); (h) dead organic matter which decomposes rapidly, humus; (s) dead organic matter in the soil, which decomposes slowly.

The magnitudes of the different reservoirs defined in this way and the present rates of exchange between them (see Figure 1) have been based primarily on the analysis by Whittaker and Likens (1975), Schlesinger (1977), and Bolin *et al.* (1979). Several of the estimates are quite uncertain and in a few instances alternate values have been used to test the sensitivity of the results to such changes. A more thorough treatment using a more detailed model is desirable.

All exchange processes within the ocean and the terrestrial biosphere will be described as first order exchange processes, while the exchange between the atmosphere and the sea and the atmosphere and the terrestrial biota will be defined more carefully, essentially following the treatment by Keeling (1973). In the case of the air-sea exchange the buffering of sea water will be included. The photosynthesis of the terrestrial biota will be dependant on both the atmospheric CO₂ concentration and the amount of photosynthesizing matter which introduces a non-linearity into the system.

We shall use the following notations.

N_i is the amount of carbon in reservoir i , where

$i = a$ denotes the atmosphere
 m mixed layer of the ocean
 d deep sea
 b short-lived biota
 w long-lived biota
 h humus
 s soil

P_m denotes the partial pressure of CO₂ in the mixed layer

F_{ij} is the annual flux from reservoir i to j

Furthermore,

$$N_i = N_{i0} + n_i \quad (2.1)$$

where N_{i0} is the steady state amount in reservoir (i) and n_i is the departure from steady state conditions. Similarly F_{ij0} denotes the steady state flux.

k_{ij} is the first order exchange coefficient.

$$k_{ij} = \frac{F_{ij0}}{N_{i0}} \quad (2.2)$$

* N_i , * N_{i0} , * n_i correspondingly, denote the amounts of radiocarbon in reservoir (i)

$$*N_i = *N_{io} + *n_i \quad (2.3)$$

$$R_i = \frac{*N_i}{N_i} ; k_i = \frac{*n}{N_i} \quad (2.4)$$

λ is the radioactive decay constant for radiocarbon.

α_{ij} is the ^{14}C fractionation for transfer from reservoir (i) to (j).

V_i is volume of the ocean reservoirs, $i = m, d$.

y_i is the annual input or removal of carbon to or from reservoir i caused by man's activities.

$*y_a$ is the annual formation of radiocarbon in the atmosphere due to cosmic radiation

The air-sea exchange requires special consideration. Following Keeling (1973) we have

$$\begin{aligned} F_{am} &= k_{am} (N_{ao} + n_a) \\ F_{ma} &= k_{am} (N_{ao} + \zeta \frac{N_{ao}}{N_{mo}} n_m) \end{aligned} \quad (2.5)$$

We shall not consider the gravitational flux from the mixed layer to the deep sea explicitly, but rather we shall follow the approach given by Keeling (1973). The coefficients, k_{md} and k_{dm} , are related by

$$k_{md} = \frac{N_{do}}{N_{mo}} \cdot k_{dm} \quad (2.6)$$

and in the equations for ^{14}C transfer a fractionation coefficient α_{dm} is introduced

$$\alpha_{dm} = \alpha_{mg} + \frac{N_{mo}}{V_m} \frac{V_d}{N_{do}} (1 - \alpha_{mg}) \quad (2.7)$$

where α_{mg} is the fractionation of ^{14}C during photosynthesis. Since α_{mg} is close to unity, the same is true for α_{dm} and no appreciable error is introduced in assuming $\alpha_{dm} = 1$.

For the photosynthesis and respiration we assume the following relations

$$\begin{aligned} F_{ab} &= F_{abo} \left(\frac{N_a}{N_{ao}} \right)^{\beta_a} \left(\frac{N_b}{N_{bo}} \right)^{\beta_b} \\ F_{ba} &= F_{bao} \frac{N_b}{N_{bo}} \end{aligned} \quad (2.8)$$

which by linearization ($n_b \ll N_{bo}$) can be transformed into

$$F_{ab} = F_{abo} \left(1 + \beta_a \frac{n_a}{N_{ao}} + \beta_b \frac{n_b}{N_{bo}} \right) \quad (2.9)$$

$$F_{ba} = F_{bao} \left(1 + \frac{n_b}{N_{bo}} \right)$$

In steady state we have, as depicted in Figure 1.

$$\begin{aligned}
 -F_{amo} - F_{abo} + F_{mao} & & + F_{bao} & & + F_{hao} & + F_{sao} & = 0 \\
 +F_{amo} & -F_{mao} - F_{mdo} + F_{dmo} & & & & & = 0 \\
 & +F_{mdo} - F_{dmo} & & & & & = 0 \\
 +F_{abo} & & -F_{bao} - F_{bwo} - F_{bho} & & & & = 0 \\
 & & +F_{bwo} & -F_{who} & & & = 0 \\
 & & & +F_{bho} + F_{who} - F_{hao} - F_{hso} & & & = 0 \\
 & & & & +F_{hso} - F_{sao} & & = 0
 \end{aligned}
 \tag{2.10}$$

For the time dependant case we get the following set of equations

$$\begin{aligned}
 \left(\frac{d}{dt} + k_{am} + \beta_a \frac{F_{abo}}{N_{ao}} \right) n_a - k_{am} \xi \frac{N_{ao}}{N_{mo}} n_m & + \left(\beta_b \frac{F_{abo}}{N_{bo}} - \frac{F_{bao}}{N_{bo}} \right) n_b & - k_{ha} n_h - k_{sa} n_s & = \gamma_a \\
 -k_{am} n_a + \left(\frac{d}{dt} + k_{am} \xi \frac{N_{ao}}{N_{mo}} + k_{md} \right) n_m - k_{dm} n_d & & & = \gamma_m \\
 & -k_{md} n_m + \left(\frac{d}{dt} + k_{dm} \right) n_d & & = \gamma_d \\
 -\beta_a \frac{F_{abo}}{N_{ao}} n_a & + \left(\frac{d}{dt} - \beta_b \frac{F_{abo}}{N_{bo}} + \frac{F_{bao}}{N_{bo}} + k_{bh} + k_{bw} \right) n_b & & = \gamma_b \\
 & & -k_{bw} n_b + \left(\frac{d}{dt} + k_{wh} \right) n_w & = \gamma_w \\
 & & -k_{bh} n_b - k_{wh} n_w + \left(\frac{d}{dt} + k_{ho} + k_{hs} \right) n_h & = \gamma_h \\
 & & & -k_{hs} n_h + \left(\frac{d}{dt} + k_{so} \right) n_s = \gamma_s
 \end{aligned}
 \tag{2.11}$$

Again following Keeling (1973), we may derive the steady state and time dependant equations for radiocarbon. In steady state we have

$$\begin{aligned}
 (\alpha_{am} F_{amo} + \alpha_{ab} F_{abo} + \lambda N_{ao}) R_{ao} - \alpha_{ma} F_{mao} R_{mo} - F_{baa} R_{bo} - F_{hao} R_{ho} - F_{sao} R_{so} &= *y_a \\
 -\alpha_{am} F_{amo} R_{ao} + (\alpha_{ma} F_{mao} + \alpha_{dm} F_{mdo} + \lambda N_{mo}) R_{mo} - F_{dmo} R_{do} &= 0 \\
 -\alpha_{dm} F_{mdo} R_{mo} + (F_{dmo} + \lambda N_{do}) R_{do} &= 0 \\
 -\alpha_{ab} F_{abo} R_{ao} + (\alpha_{ba} F_{baa} + F_{bwo} + F_{bho} + \lambda N_{bo}) R_{bo} &= 0 \\
 -F_{bwo} R_{bo} + (F_{who} + \lambda N_{wo}) R_{wo} &= 0 \\
 -F_{bho} R_{bo} - F_{who} R_{wo} + (F_{hao} + F_{hso} + \lambda N_{ho}) R_{ho} &= 0 \\
 -F_{hso} R_{ho} + (F_{sao} + \lambda N_{so}) R_{so} &= 0
 \end{aligned}
 \tag{2.12}$$

Summation yields

$$\lambda \Sigma N_{io} R_{io} = *y_a \tag{2.13}$$

The numerical values for the fractionation factors α_{ij} were given by Keeling (1973). No fractionation within the terrestrial biota is assumed to exist, but is obviously easy to account for if required.

$$*N_{ij} \cdot *n_i = K_i \tag{2.14}$$

where

3. STEADY STATES, PRESENT AND PAST

The systems of equations (2.10) and (2.12) define the steady state distribution of ^{12}C and ^{14}C between the seven reservoirs, in the latter case in response to a constant rate of ^{14}C production, $*\gamma_a$. The former system, (2.10), essentially only implies a consistency in the estimates from observations of the fluxes between the reservoirs. The exceptions are the air sea exchange and the exchange within the oceans where the ^{14}C -distribution has been utilized as well (Craig, 1957; see further below). With the aid of (2.2) and the estimates of the reservoir sizes we also obtain a value for the exchange coefficients, k_{ij} , required for the maintenance of a steady state. The estimate of F_{dmo} and thus of k_{dm} is based on the value of R_{do} , the third equation (2.12) and the assumption that $F_{\text{mdo}} = F_{\text{dmo}}$. A deviation of real conditions from a steady state may imply a considerable error in the estimate of k_{dm} . ^{14}C observations from the deep sea (R_{d}) are available for merely about 20 years, which is much too short a time to reveal possible deviations from a steady state.

Most simulations of future changes of the carbon cycle have been based on an assumption that an approximate steady state prevailed at the beginning of the industrial revolution in the middle of the last century and the determination of some features of the cycle by asking for the best possible fit between the model and reality during the rather short period for which data are available. The assumption of an initial steady state is crucial for this approach. Because we do not know how accurate this approximation is, a considerable degree of uncertainty is introduced that has usually not been taken into account. We shall later explicitly investigate the implications, if this assumption is not valid. In the remainder of this section some other possible steady states will be discussed.

Comparisons between radiocarbon age and real age of tree rings show considerable differences (Suess, 1970) with variations on time scales of a few hundred years. The radiocarbon age also becomes increasingly less than the real age when going back 4.000 to 6.000 years in time. The maximum difference is about 900 years and has been explained as a result of a more intense magnetic field, whereby the production of ^{14}C by cosmic rays, $*\gamma_a$, may have been greater. This explanation seems plausible, since there is independent evidence that such changes may have occurred in the past (Bucha, 1970).

It is, however, interesting to note that measurements of the decrease of the ^{14}C with depth in the bottom sediments, even if accounting for this possible change of $*\gamma_a$, show a decrease of the sedimentation rate from 10.000 B.P. until present times (Peng *et al.*, 1977). The rate of primary production in the sea therefore probably was less during this time period. Since the primary production is dependant on the availability of nutrients in the photic zone and since these are supplied by up-welling one may ask if possibly the turn-over rate of the oceans may have been different at that time. In this context it should be recalled that Worthington (1968) has advanced the idea that the deglaciation about 10.000 years ago may have decreased the salinity of the polar waters to such an extent that the deep water formation may have been con-

siderably reduced or even completely inhibited. This reduction would then, however, also have changed the distribution of radiocarbon. Obviously a decrease of the deep water formation would imply less radiocarbon in the deep sea, but also larger amounts in the other major reservoirs if the production rate, $*y_a$, remained the same. Let us assume that at any one time an approximate steady state prevailed and determine the ^{14}C in the various reservoirs for different values of $k_{\text{dmo}} = F_{\text{dmo}}/N_{\text{do}}$, for the present value of $*y_a$.

It was shown by Keeling (1973) that the present simple formulation of the carbon exchange in the oceans without considering the biological activity explicitly is a reasonable approximation if a steady state prevails. We use the reservoir sizes and exchange rates as given in Figure 1, but vary k_{dmo} . The fractionation factors given by Keeling (1973) have been used. Figure 2 shows how the radiocarbon concentration of the atmosphere and the two ocean reservoirs depend on k_{dmo} . We denote the new values with R'_{i0} and Figure 2 shows $R'_{\text{ao}}/R_{\text{ao}}$, $R'_{\text{mo}}/R_{\text{ao}}$ and $R'_{\text{do}}/R_{\text{ao}}$, where R_{ao} , denote the values for $k_{\text{dm}}^{-1} = 1.000$ years. Since no changes of the behaviour of the terrestrial biosphere have been assumed to occur, $R'_{\text{bo}}/R_{\text{ao}}$, $R'_{\text{wo}}/R_{\text{ao}}$, $R'_{\text{ho}}/R_{\text{ao}}$ and $R'_{\text{so}}/R_{\text{ao}}$ change directly in proportion to $R'_{\text{ao}}/R_{\text{ao}}$.

We note that R'_{ao} and R'_{mo} change considerably more than R'_{do} depending on the much larger deep ocean water reservoir. R'_{ao} (and R'_{mo}) increase by 10% if k_{dm}^{-1} increases to about twice the present value, i.e. to about 2.000 years. This change would be sufficient to explain the discrepancy of 900 years between radiocarbon age and true age which has been observed for tree rings grown 6.000 years ago, without

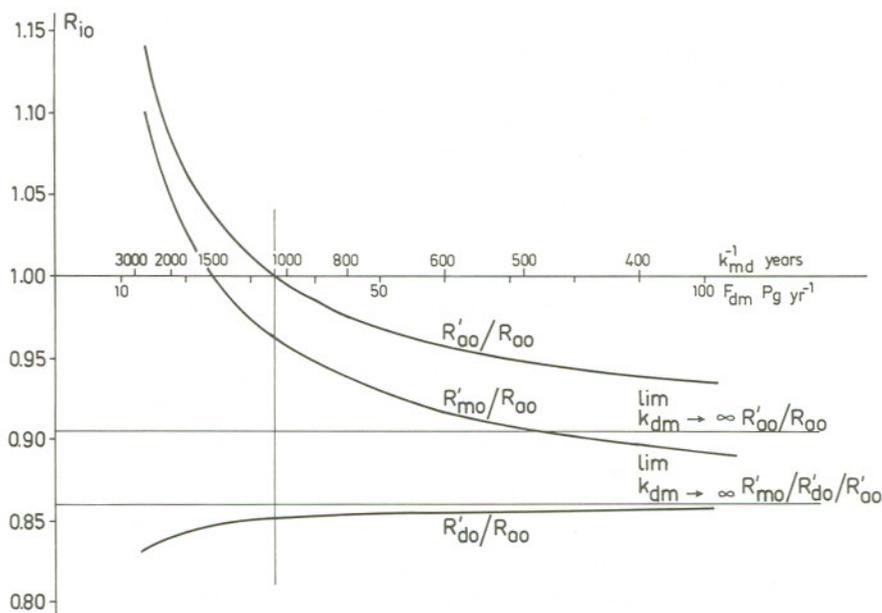


Figure 2. R'_{i0} ($i = a, m, d$) as function of the rate of exchange between the deep sea and the mixed layer, F_{dm} , or the turn-over time, $\tau_{\text{dm}} = k_{\text{dm}}^{-1}$.

any change of the radio-carbon production by cosmic radiation. It is interesting to note that the sedimentation for the period 10.000–6.000 B.P. that has been computed by Peng *et al.* (1977) are about 40% of the present rate. This value is in reasonable agreement with the notion that the rate of up-welling, nutrient supply to the photic zone and primary production is proportional to k_{dmo} and thus 50% of the present value (if $k_{\text{dm}}^{-1} = 2.000$ years). It should be remarked, however, that Peng *et al.* (1977) seems not to have corrected their estimates of sedimentation rates by considering also the difference between radiocarbon age and true age.

The computations summarized above are approximate and the data used are quite limited. We therefore cannot conclude that the ocean circulation has varied during postglacial time, even though the data referred to support such a conclusion. In view of its important implications for the determination of past changes of climate the problem warrants further attention.

4. TRANSIENT CHANGES IN THE CARBON CYCLE

The two sets of equations (2.11) and (2.14) describe the transient behaviour of the carbon cycle due to external factors that influence the exchange of ^{12}C and thereby indirectly also the distribution of ^{14}C . We shall in turn consider

- 1) The eigencharacteristics of the systems
- 2) The adjustment in time of an initial departure from equilibrium
- 3) The response of the carbon cycle to an exponentially increasing source term γ_a for different rates of increase
- 4) The response of the carbon cycle to a periodic variation of the surface water temperature

4.1 THE EIGENCHARACTERISTICS OF THE CARBON CYCLE

The eigenfrequencies of the system (2.9) with the values of F_{ij_0} and N_{io} (and thus k_{ij}) as given in section 2 are ($\beta_a = 0.10$)

$v_1 = -1.17 \text{ yr}^{-1}$	$T_1 = 0.85 \text{ yr}$
$v_2 = -0.91 \text{ yr}^{-1}$	$T_2 = 1.09 \text{ yr}$
$v_{3,4} = -0.0210 \pm i 0.042 \text{ yr}^{-1}$	$T_{3,4} = 48 \pm i 150 \text{ yr}$
$v_4 = -0.0034 \text{ yr}^{-1}$	$T_5 = 300 \text{ yr}$
$v_6 = -0.00092 \text{ yr}^{-1}$	$T_6 = 1090 \text{ yr}$
$v_7 = 0$	$T_7 = \infty$

The range of v_i is the same as for k_{ij} . We note further that there is one pair of complex eigenfrequencies permitting damped oscillations (see 4.2). This appears because of the time delay of transfer through the terrestrial biota. Other values β_a ($0.01 \leq \beta_a \leq 0.20$) do not greatly influence the principle features as revealed by the eigenfrequencies given for $\beta_a = 0.10$. It should be emphasized, however, that the nonlinear equations that describe finite changes in the system have been linearized, whereby a more

complex behaviour of the system has been possibly eliminated. For a more detailed analysis of such a possibility reference is made to an early paper by Eriksson and Welander (1956).

4.2 THE ADJUSTMENT OF AN INITIAL DEPARTURE FROM EQUILIBRIUM

The carbon cycle has been significantly influenced by man during at least 200 years, first by reduction of forest extension and an increase of farm land, during the last 100 years also by emissions of carbon dioxide to the atmosphere due to fossil fuel combustion. We do not therefore know how well an equilibrium prevailed at any one time during the last two centuries and it is of interest to analyze the way in which a departure from equilibrium gradually is modified by internal adjustments.

Figure 3a shows how an initial excess of 10 Pg in the atmosphere is distributed among the other reservoirs during the following one hundred years. After one century there are still considerable departures from equilibrium and these prevail for a long time thereafter. The rates of change are still quite large after 40–50 years. The changes that are being observed during a given year ("air-borne fraction") must therefore not only be related to emissions during that particular year but in reality may to a considerable extent be adjustments to disturbances of the equilibrium that occurred several decades earlier. Naturally the adjustment process as shown in

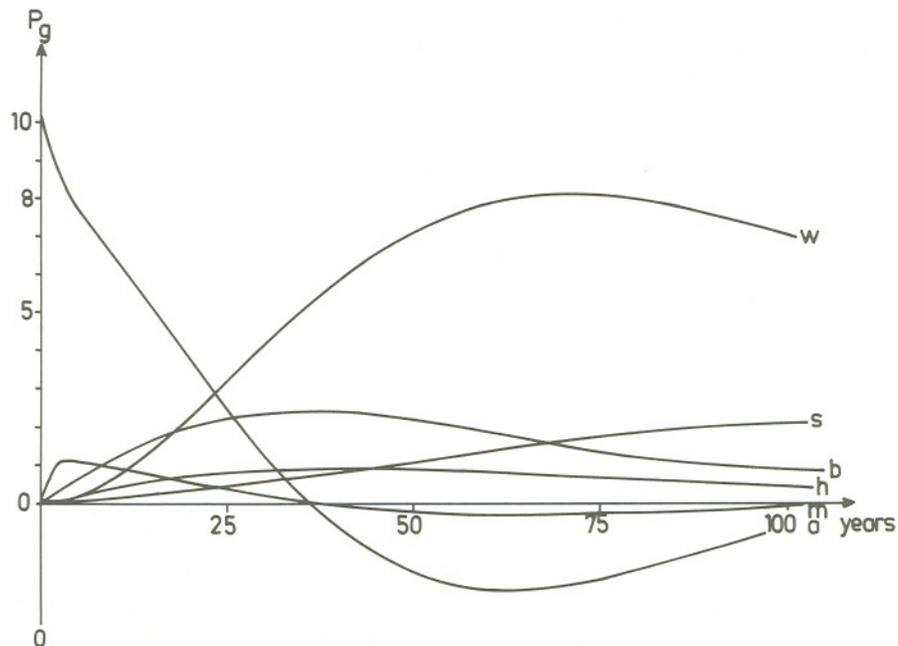


Figure 3 a

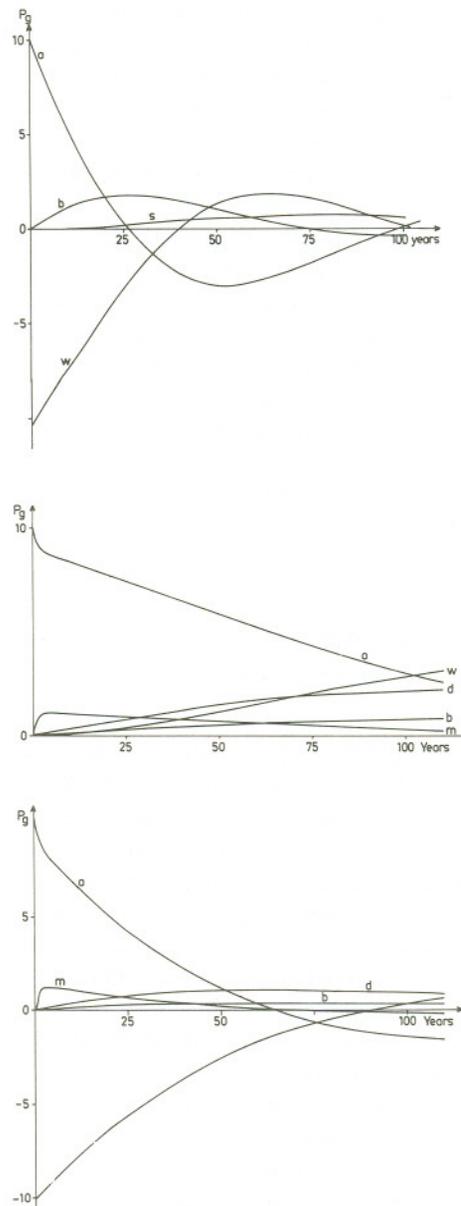


Figure 3. The adjustments of various reservoirs following a disturbance of
 a) the addition of 10 Pg to the atmosphere, $\beta = 0.10$.
 b) the addition of 10 Pg to the atmosphere and withdrawal of 10 Pg from long-lived biota, i.e. wood, $\beta = 0.10$.
 c) the same as a) but $\beta = 0.01$.
 d) the same as b) but $\beta = 0.01$.

Figure 3a is critically dependant on the parameters chosen to describe the carbon cycle. The marked increase of n_w is a response to the increase of n_b , which in turn depends on the value chosen for β_a . A smaller value for β_a delays the decrease of n_a very considerably, whereby n_m also remains positive longer permitting a build up of n_d (see Figure 3c). Most of the excess carbon injected into the atmosphere in any case ultimately ends up in the deep sea, but for a small value of β_a much less passes via the terrestrial biota as illustrated in Figure 3a.

Another example is shown in Figure 3b in which the initial disturbances are $n_{a0} = 10$ Pg and $n_{w0} = -10$ Pg, i.e. no net change of the total amount of carbon in the system is introduced. Such a situation would result from the burning of forests. We note that adjustments back to a steady state are quicker than in the previous example, because the reservoirs with slow response, i.e. the deep sea and the soil, never get much involved in this case. Again a small value for β_a delays the changes as shown in Figure 3d. However in simulating the expansion of agriculture by putting $n_{a0} = 10$ Pg and $n_{s0} = -10$ Pg, we find much more long lasting effects on the carbon cycle because of the slow turn-over time for carbon in soils.

4.3 THE RESPONSE TO AN EXPONENTIALLY INCREASING SOURCE TERM y_a .

The emission of carbon dioxide to the atmosphere by fossil fuel combustion during the last 100 years can be approximated reasonably well by an exponential function $y = y_0 \exp(\alpha t)$, where $\alpha = 0.03 \text{ years}^{-1}$. (The increase was significantly lower during the two world wars and also during the economic depression in the beginning of the 1930's.) The response of the carbon cycle to such an increase may be considered as the sum of a forced exponential solution to equations (2.11) and adjustments (solutions to the homogenous equations (2.11)), which tend towards zero as time becomes large compared with the periods defined by the eigenfrequencies of the system. The character of the adjustments is determined by the initial conditions, and thus depends also on whether the system initially is in equilibrium or not. For a detailed comparison between the model and reality the complete solution of course must be considered, which, however, is uncertain if initial conditions are not well known. It is of interest to have a look at merely the forced solution since it well reveals the general behaviour of the system, particularly its dependance on the rate with which the external influence is imposed on the system.

With $y_a = y_{a0} \exp(\alpha t)$ the forced (particular) solution of equations (2.11) is given by

$$\begin{aligned} n_i &= n_{i0} \exp(\alpha t) \\ \Sigma n_{i0} &= y_{a0}/\alpha \end{aligned} \quad (4.1)$$

Table 1 shows n_{i0} , $i = a, m, \dots$ in percent of y_{a0}/α for values of $\alpha = 0.5, 1, 2 \dots 8\%$

Table 1. $n_{i0}/(y_{a0}/\alpha)$ in percent for $i = a, m, d \dots s$, for different rates of the emission increase, α .

n_{i0}	$\alpha\%$						
y_{a0}/α	0.5	1.0	2.0	3.0	4.0	6.0	8.0
$i = a$	3.7	8.7	19.5	29.5	38.0	50.6	59.0
m	0.5	1.2	2.7	4.0	5.1	6.6	7.6
d	2.8	3.6	4.2	4.3	4.1	3.6	3.1
b	11.5	13.7	15.4	15.5	14.9	13.3	11.6
h	5.0	5.7	6.0	5.8	5.4	4.6	3.9
w	50.0	50.3	42.8	34.8	28.0	18.8	13.2
s	26.4	16.8	9.4	6.2	4.4	2.5	1.6

We note that the partitioning between the various reservoirs very much depends on the rate of increase, α . For small values of α the uptake by the oceans is small, while the more accessible reservoirs of terrestrial biota and soil are the important sinks. The result is, however, sensitive to the choice of β_a and β_b as is shown by Kohlmaier (chapter 5, this volume). Particularly for values $\beta_b > 1$ the biota quickly decreases in importance as a sink (cf the behaviour of the solutions presented by Revelle and Munk, 1977).

4.4 THE RESPONSE OF THE CARBON CYCLE TO A PERIODIC VARIATION OF THE SURFACE WATER TEMPERATURE

Bacastow (1978) and MacIntyre (1978) have analyzed to what extent temperature changes of the surface water may cause variations of the carbon dioxide concentrations in the atmosphere. MacIntyre points out the importance of considering the buffering characteristics of sea water for properly evaluating the effect of a temperature change on the equilibrium partial pressure. Let P denote the partial pressure, T the temperature, and C the concentration of total inorganic carbon in sea water, while Σ denotes the total amount of carbon in the gas phase and in solution. MacIntyre (1978) emphasizes that $(\delta P/\delta T)_c \approx 16-18$ ppm/ $^{\circ}\text{C}$ is very much larger than $(\delta P/\delta T)_{\Sigma} = 1.3-1.5$ ppm/ $^{\circ}\text{C}$ and that the temperature effect in reality is therefore much reduced. Bacastow (1978) analyzes the implications of this in a simple model of the carbon cycle and shows that the carbon dioxide variations observed in association with the Southern Oscillation could be explained by a temperature variation of about $\pm 1^{\circ}\text{C}$.

It is of interest in the present context to analyze the effect of temperature variations of different frequencies for the somewhat more general carbon cycle used here. Following Bacastow (1978) we have

$$\delta P_m = \left(\frac{\delta P_m}{\delta C_m} \right)_T \delta C_m + \left(\frac{\delta P_m}{\delta T} \right)_{C_m} \delta T \quad (4.2)$$

where index $()_m$ as before denotes the mixed layer and δ indicates small deviations from equilibrium. From Keeling (1973) and Bacastow (1978) we get

$$\delta F_{ma} = k_{am} N_{ao} \frac{\delta P_m}{P_m} = k_{am} N_{ao} \xi \frac{\delta N_m}{N_{mo}} + \frac{1}{P_m} \left(\frac{\delta P_m}{\delta T} \right)_{C_m} \delta T \quad (4.3)$$

The first term has been included in (2.11) ($\delta N_m \approx n_m$ for small n_m), and we can account for the influence of a varying temperature on the changes of n_i by putting

$$\gamma_a = k_{am} N_{ao} \frac{1}{P_m} \left(\frac{\delta P_m}{\delta T} \right)_{C_m} \delta T = -\gamma_m \quad (4.4)$$

Assuming $P_m^{-1} (\delta P_m / \delta T)_{C_m} = 0.04$ we obtain

$$\gamma_a = 4 \cdot \delta T = -\gamma_m \quad (4.5)$$

We insert the expression (4.4) with the numerical value given in (4.5) into (2.11) and assume a periodic variation of δT with different frequencies. The amplitude of the associated periodic variations of n_a are shown in Figure 4.

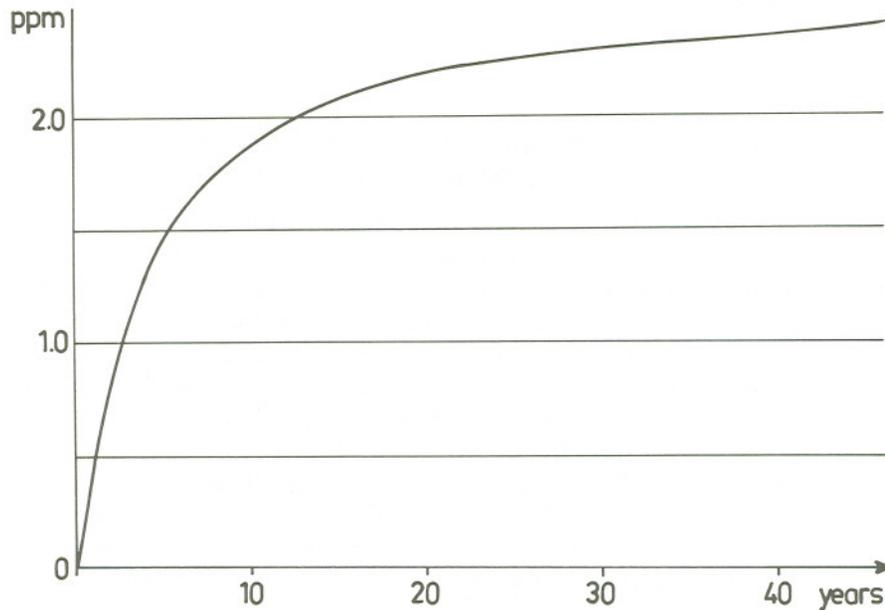


Figure 4. The amplitude of the variations of the atmospheric concentration (parts per million) of carbon (in the form of carbon dioxide) due to a periodic change of the temperature of the mixed layer with an amplitude of 1°C and as dependant on the period of the forcing.

For a period $\tau = 4$ years, which corresponds to the period of the Southern Oscillation, the result agrees well with that of Bacastow (1978). Since the response depends primarily on the rate of exchange between the atmosphere and the mixed layer of the sea, which is rapid, an approximate equilibrium between these two reservoirs prevails at anyone time for variations with a period of 5–10 years and longer. No further appreciable increase of the amplitude of n_a occurs for larger values of τ . For quick temperature changes, however, there is not enough time to have a quasi-equilibrium established; the amplitude of n_a is correspondingly less and the phase lags behind that of δT . For annual variations it is merely 0.4 ppm. In reality, therefore, little of the annual variations of n_a are probably caused by the annual variations of ocean temperature.

REFERENCES

- Bacastow, R.B. (1978) Dip in the atmospheric CO₂ level during the mid 1960's. *J. Geoph. Res.*
- Björkström, A. (1979) A model of the CO₂ interaction between atmosphere, oceans and land biota. In Bolin, B., Degens, E.T., Kempe, S., Ketner, P. (eds), *The global carbon cycle*, SCOPE Report, 13, 403–457, J. Wiley & Sons, Chichester.
- Bolin, B., Degens, E.T., Duvigneaud, P. and Kempe, S. (1979) The global biogeochemical cycle of carbon. In Bolin, B., Degens, E.T., Kempe, S., Ketner, P. (eds), *The global carbon cycle*, SCOPE Report, 13, 1–56, J. Wiley & Sons, Chichester.
- Bucha, V. (1970) Evidence for changes in the Earth's magnetic field intensity. *Phil. Trans. Roy. Soc. A* 269, 47–55.
- Craig, H. (1957) The natural distribution of radiocarbon and the exchange time of carbon dioxide between atmosphere and sea. *Tellus*, 9, 1–17.
- Eriksson, E. (1963) Possible fluctuations in atmospheric CO₂ due to changes in the properties of the sea. *J. Geoph. Res.*, 68, 3871–3876.
- Eriksson, E. and Welander, P. (1956) On a mathematical model of the carbon cycle in nature. *Tellus*, 8, 155–175.
- Keeling, C.D. (1973) The carbon dioxide cycle: Reservoir models to depict the exchange of atmospheric carbon dioxide with the oceans and land plants. In: Rasool, S.I. (ed), *Chemistry of the lower atmosphere*, 251–329, Plenum Press, New York.
- MacIntyre, F. (1978) On the temperature coefficient of P_{CO₂} in sea water. *Climatic change*, 1, 349–354.
- Peng, T.-H., Broecker, W.S., Kipphut, G. and Shackleton, N. (1977) Benthic mixing in deep sea cores as determined by ¹⁴C dating and its implications regarding climate stratigraphy and the fate of fossil fuel CO₂. In: Andersson, N., Malahoff, A. (eds) *The fate of fossil fuel CO₂ in the oceans*, 355–373. Plenum Press, New York.
- Revelle, R., Munk, W. (1977) The carbon dioxide cycle and the biosphere. In: *Energy and climate*. Stud. Geophys. 140–158. Nat. Acad. of Sciences, Washington, D.C.
- Schlesinger, W.H. (1977) Carbon balance in terrestrial detritus. *Am. Rev. Ecol. Syst.* 51–81.
- Suess, H. (1970) The three causes of the secular C14 fluctuations and their time constants. In: Olsson, I. (ed) *Radiocarbon variations and absolute chronology*, 595–605, Proc. Twelfth Nobel Symp. Almqvist & Wicksell, Uppsala.
- Whittaker, R.H. and Likens, G.E. (1975) The biosphere and man. In "Primary productivity of the biosphere". *Ecological Studies*, 14, 305–328, Springer Verlag, Berlin.
- Worthington, L.V. (1968) Genesis and evolution of water masses, *Meteorological monographs*, Am. Met. Soc. Boston, Mass., Vol. 8, 63–67.

