Hemispheric Airborne Fractions Difference and the Hemispheric Exchange Time

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The observed airborne fraction at the South Pole, 0.51, is significantly less than the observed airborne fraction at Mauna Loa, 0.55 (see chapter 4). This is probably a consequence of the industrial dominance of the Northern Hemisphere and an equatorial barrier to CO_2 transport. Only approximately 4.3% of fossil fuel is produced, and, we assume, consumed in the Southern Hemisphere.

A simple box model (figure 1) in which the atmosphere and surface ocean are divided at the equator illustrates how a difference in airborne fractions may arise. The effect of movement of surface ocean water across the equator will be neglected because it is relatively small, and the coupling of the surface ocean layer to the deeper waters will be neglected because the time scale of interest is only about 20 years. Material balances on the four boxes lead to the following equations:

$$\frac{dx_1}{dt} = -k(X_{10} + x_1) + k(X_{20} + x_2)$$
$$-k_{am}(X_{10} + x_1) + k_{ma}(Z_{10} + \xi z_1) + y_1(t) \tag{1}$$

$$\frac{dx_2}{dt} = -k(X_{20} + x_2) + k(X_{10} + x_1)$$
$$-k_{am}(X_{20} + x_2) + k_{ma}(Z_{20} + \xi z_2) + y_2(t) \tag{2}$$

$$\frac{dz_1}{dt} = k_{am}(X_{10} + x_1) - k_{ma}(Z_{10} + \xi z_1) \tag{3}$$

$$\frac{dz_2}{dt} = k_{am}(X_{20} + x_2) - k_{ma}(Z_{20} + \xi z_2) \tag{4}$$
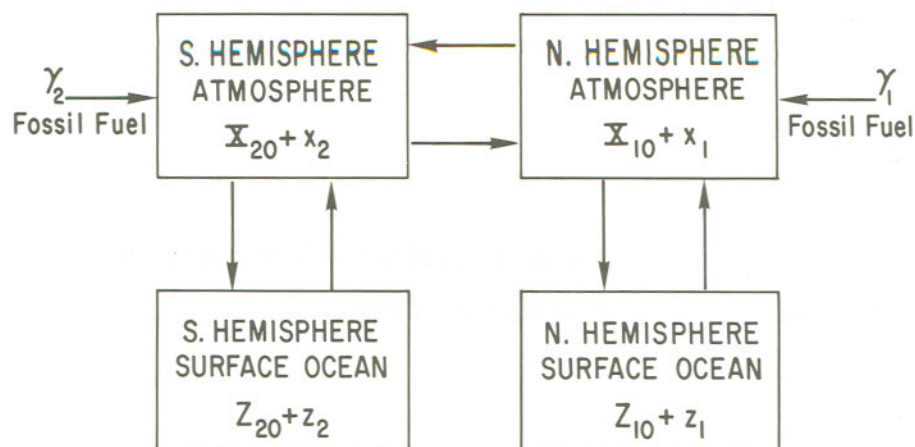
Figure 1. Hemispheric model used to estimate atmospheric exchange time between hemispheres.

where we have introduced the symbols:

X_{10}, X_{20} preindustrial steady-state carbon abundances in the atmospheric Northern and Southern Hemisphere, respectively (gigaton, gt).

Z_{10}, Z_{20} preindustrial steady state carbon abundances in the surface ocean Northern and Southern Hemispheres, respectively (gt).

x_1, x_2, z_1, z_2 perturbations in X_{10}, X_{20}, Z_{10} and Z_{20}, respectively (gt).

t time (yr).

k hemispheric exchange constant (yr^{-1}).

k_{am} atmosphere to surface ocean exchange constant (yr^{-1}).

k_{ma} surface ocean to atmosphere exchange constant (yr^{-1}).

ξ surface ocean CO_2 buffer factor.

With the assumption of an exponential total fossil fuel source to the atmosphere of $y_o e^{\mu t}$ gt yr^{-1}, the separate hemispheric sources are

$$y_1 = (1 - 0.043)\, y_o e^{\mu t} \tag{5}$$

and

$$y_2 = 0.043\, y_o e^{\mu t} \tag{6}$$

The steady state equations (obtained by setting the derivatives and perturbations to zero) can be subtracted from the above equations, resulting in equations involving only the perturbations:

$$\frac{dx_1}{dt} = -k(x_1 - x_2) - k_{am} x_1 + k_{ma} \xi z_1 + y_1 \tag{7}$$

$$\frac{dx_2}{dt} = k(x_1 - x_2) - k_{am} x_2 + k_{ma} \xi z_2 + y_2$$

$$\frac{dz_1}{dt} = k_{am} x_1 - k_{ma} \xi z_1 \tag{9}$$

$$\frac{dz_2}{dt} = k_{am} x_2 - k_{ma} \xi z_2 \tag{10}$$

To facilitate the solution of these equations for the difference in airborne fractions, we introduce new variables defined as the sums and differences of the atmospheric and oceanic perturbations:

$$y_1 \equiv x_1 - x_2 \tag{11}$$

$$y_2 \equiv x_1 + x_2 \tag{12}$$

$$y_3 \equiv z_1 - z_2 \tag{13}$$

$$y_4 \equiv z_1 + z_2 \tag{14}$$

After (8) is subtracted from (7) and (10) from (9),

$$\frac{dy_1}{dt} = -(2k + k_{am}) y_1 + k_{ma} \xi y_3 + f y_o e^{\mu t} \tag{15}$$

and

$$\frac{dy_3}{dt} = k_{am} y_1 - k_{ma} \xi y_3 \tag{16}$$

where

$$f = 1 - 0.043 - 0.043 = 0.914 \tag{17}$$

Two other independent equations can be found by adding (7) and (8), and (9) and (10), but they will not be needed. The solution to (15) and (16) will be a sum of exponential terms with time constants λ_1, λ_2, and μ, but only the term with exponential time constant μ will be important:

$$\lambda_1, \lambda_2 = \frac{2k + k_{ma} \xi + k_{am}}{2} \left[-1 \pm \sqrt{1 - \frac{8 k k_{ma} \xi}{(2 k + k_{ma} \xi + k_{am})^2}} \right] \tag{18}$$

$$\approx k \left[-1 \pm (1 - \frac{k_{ma} \xi}{k})\right] = \begin{cases} -2k + k_{ma} \xi \\ -k_{ma} \xi \end{cases} \tag{19}$$

The approximate solution for the roots follows from a consideration of the relative magnitude of the time constants:

$$1/k \approx 1 \text{ yr} \tag{20}$$

$$1/k_{am} \approx 7.5 \text{ yr} \tag{21}$$

$$\xi \approx 10 \tag{22}$$

$$k_{ma} \approx \frac{X_{10} + X_{20}}{Z_{10} + Z_{20}} \; k_{am} \approx \frac{76}{300} \; k_{am} \approx 1/29.6 \tag{23}$$

where (23) follows from the sum of the steady state parts of (1) and (2), and we take the surface ocean layer to be 300 m deep (so as to approximately include the effect of water below the ~75 m mixed surface layer) and use the approximation that the amount of carbon in the atmosphere is equivalent to th carbon in 76 m of ocean surface water. Then for times much greater than $1/(k_{ma} \, \xi) = 3.0$ years, only the term exponential with time constant $1/\mu$ is important. The solution then can be gotten directly by assuming all the perturbations grow exponentially as $e^{\mu t}$, i.e.:

$$y_1 = \tilde{y}_1 \, e^{\mu t} \tag{24}$$

$$y_3 = \tilde{y}_3 \, e^{\mu t} \tag{25}$$

and (15) and (16) become

$$\mu \, \tilde{y}_1 = -(2 \, k + k_{am}) \, \tilde{y}_1 + k_{ma} \, \xi \, \tilde{y}_3 + f \, y_o \tag{26}$$

and

$$\mu \, \tilde{y}_3 = k_{am} \, \tilde{y}_1 - k_{ma} \, \xi \, \tilde{y}_3 \tag{27}$$

The solution for y_1 is then

$$y_1 = \frac{f \, y_o}{2 \, k + \mu \, \left(1 + \dfrac{k_{am}}{\mu + k_{ma} \, \xi}\right)} \; e^{\mu t} \tag{28}$$

During the Mauna Loa and South Pole records, the increase in y_1 is from $y_1 \, (t_1)$ to $y_1 \, (t_2)$:

$$\Delta y_1 = y_1 \, (t_2) - y_1 \, (t_1) = \frac{f \, y_o}{2 \, k + \mu \, \left(1 + \dfrac{k_{am}}{\mu + k_{ma} \, \xi}\right)} \; (e^{\mu t2} - e^{\mu t1}) \tag{29}$$

Fossil fuel input to the atmosphere during the same period is

$$\Delta F = \int_{t_1}^{t_2} y_o \, e^{\mu}t = \frac{y_o}{\mu} \, (e^{\mu t2} - e^{\mu t1}) \tag{30}$$

The ratio

$$\frac{\Delta y_1}{\Delta F} = \frac{f\mu}{2k + \mu \left(1 + \dfrac{k_{am}}{\mu + k_{ma}\,\xi}\right)} \tag{31}$$

is 1/2 the difference in apparent airborne fractions:

$$\frac{\Delta y_1}{\Delta F} = \frac{[x_1(t_2) - x_1(t_1)] - [x_2(t_2) - x_2(t_1)]}{\Delta F}$$

$$= \frac{[x_1(t_2) - x_2(t_2)] - [x_1(t_1) - x_2(t_1)]}{\Delta F}$$

$$= 1/2\ \Delta ABF = \frac{1}{2}\left(\frac{18.80 - 17.31}{34.19}\right) = 0.0218 \tag{32}$$

where the numbers come directly from table 7 of Bacastow and Keeling, chapter 4.

The exchange time between the hemispheres, 1/k, can be estimated from (31) and (32):

$$1/k = \frac{2/\mu}{\dfrac{f}{1/2\ \Delta ABF} - 1 - \dfrac{k_{am}}{\mu + k_{ma}\,\xi}} \tag{33}$$

$$= \frac{(2)(23)}{\dfrac{0.914}{0.0218} - 1 - \dfrac{1/7.5}{\dfrac{1}{23} + \dfrac{76}{300}\dfrac{10}{7.5}}} = 1.14\ \text{yr} \tag{34}$$

From (34) it is apparent that the exchange time is not sensitive to the assumed depth of the surface layer (300 m). The difference in increase between Mauna Loa and the South Pole is $18.80 - 17.31 = 1.49$ ppm, for which we estimate an error of ± 0.2 ppm, hence an error on the exchange time of ± 0.16 yr. The method used here to estimate k depends only on the relative increase in each hemisphere, and not directly on the absolute value of the difference in concentration between hemispheres.

The value of k found above is in excellent agreement with an estimate of 0.8 ± 0.1 yr^{-1} based on the hemispheric differences in the essentially inert chlorofluorocarbon gases CCl_3F and CCl_2F_2 (F.S. Rowland, personal communication). It is in reasonable agreement with an estimate of Newell *et al.* (1968) of $1/k = 0.88$ yr from meteorological data in the tropics. Other estimates are reviewed by Czeplak and Junge (1974), who show the two box hemisphere model to be a good approximation to the more realistic eddy diffusive model for temperate injections of a gas with a residence time longer than a few months.

The good agreement in our estimate of k and the two other estimates above can be interpreted as an indication that the biospheric source/sink in the two hemispheres are equal in size within, unfortunately, rather large limits. To show how this conclusion arises, we will attribute the difference between our estimate of 1/k and that of Newell *et al.* to the effect of the land biosphere. Biosphere sources in the Northern and Southern Hemispheres of $g_1 y_o e^{\mu t}$ and $g_2 y_o e^{\mu t}$, respectively, result in the replacement of f by f + g in (15), (26), (28), (29), (31) and (33), where

$$g \equiv g_1 - g_2 \tag{36}$$

We then find g = 0.25. This is an algebraic difference; it would imply that either the Northern Hemisphere was a larger source than the Southern Hemisphere, or the Southern Hemisphere is a larger sink than the Northern Hemisphere, or some combination of the two. We need more information or an additional assumption to go further.

Junge and Czeplak (1965) estimate that carbon uptakes by plants in the Northern and Southern Hemispheres are 22.6 and 15.8 gt yr^{-1}, respectively. Let us assume that the hemispheres are both either sources or sinks, and that the sizes of those sources or sinks are proportional to the Junge-Czeplak numbers above, i.e.:

$$\frac{g_1}{22.6} = \frac{g_2}{15.8} \tag{37}$$

Then

$$g_1 \left(1 - \frac{15.8}{22.6}\right) = 0.25 \tag{38}$$

and the solution is $g_1 = 0.83$ and $g_2 = 0.58$. The biosperic source would be

$$(0.86 + 0.60) \, (5.6 \text{ gt/year}) = 8.2 \text{ gt/year} \tag{39}$$

With longer time series, the errors on the estimates of k from CO_2 data and from other methods should decrease and then a more useful estimate of the biospheric source/sink may be obtained by the method used above. Continued exponential growth in the use of fossil fuels is not required for this analysis; a more complicated analysis could use observed fossil fuel production data.

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